

Problem 30

In each of Problems 30 through 33, find the Laplace transform of the given function. In Problem 33, assume that term-by-term integration of the infinite series is permissible.

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

Solution

The Laplace transform of a function $f(t)$ is defined to be

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt.$$

Of course, for this integral to converge, it's necessary that $s > 0$. Split up the integral over the intervals that the given function is defined over.

$$\begin{aligned} F(s) &= \int_0^1 e^{-st}(1) dt + \int_1^{\infty} e^{-st}(0) dt \\ &= \int_0^1 e^{-st} dt \\ &= -\frac{1}{s} e^{-st} \Big|_0^1 \\ &= \frac{1}{s} - \frac{1}{s} e^{-s} \\ &= \frac{1 - e^{-s}}{s} \end{aligned}$$