Problem 4

In each of Problems 4 through 7, find the Laplace transform of the given function.

$$f(t) = \int_0^t (t-\tau)^2 \cos 2\tau \, d\tau$$

Solution

Recognize that $f(t)$ is a convolution integral of the two functions, t and $\cos 2t$. The Laplace transform of $f(t)$ can be found by using the convolution theorem, which states that

$$\mathcal{L}\left\{\int_0^t g(t-\tau)h(\tau)\,d\tau\right\} = G(s)H(s),$$

where G and H are the Laplace transforms of g and h, respectively. Therefore,

$$\begin{aligned}
\mathcal{L}\{f(t)\} &= \mathcal{L}\left\{\int_0^t (t-\tau)^2 \cos 2\tau \, d\tau\right\} \\
&= \mathcal{L}\{t^2\}\mathcal{L}\{\cos 2t\} \\
&= \left(\frac{2!}{s^{2+1}}\right)\left(\frac{s}{s^2+4}\right) \\
&= \frac{2s}{s^3(s^2+4)} \\
&= \frac{2}{s^2(s^2+4)}.
\end{aligned}$$