

## Problem 7

In each of Problems 4 through 7, find the Laplace transform of the given function.

$$f(t) = \int_0^t \sin(t - \tau) \cos \tau \, d\tau$$

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### Solution

Recognize that  $f(t)$  is a convolution integral of the two functions,  $\sin t$  and  $\cos t$ . The Laplace transform of  $f(t)$  can be found by using the convolution theorem, which states that

$$\mathcal{L} \left\{ \int_0^t g(t - \tau)h(\tau) \, d\tau \right\} = G(s)H(s),$$

where  $G$  and  $H$  are the Laplace transforms of  $g$  and  $h$ , respectively. Therefore,

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \mathcal{L} \left\{ \int_0^t \sin(t - \tau) \cos \tau \, d\tau \right\} \\ &= \mathcal{L}\{\sin t\} \mathcal{L}\{\cos t\} \\ &= \left( \frac{1}{s^2 + 1} \right) \left( \frac{s}{s^2 + 1} \right) \\ &= \frac{s}{(s^2 + 1)^2}. \end{aligned}$$