Problem 21

Consider the equation

\[ \phi(t) + \int_0^t k(t - \xi)\phi(\xi) \, d\xi = f(t), \]

in which \( f \) and \( k \) are known functions, and \( \phi \) is to be determined. Since the unknown function \( \phi \) appears under an integral sign, the given equation is called an integral equation; in particular, it belongs to a class of integral equations known as Volterra integral equations. Take the Laplace transform of the given integral equation and obtain an expression for \( L\{\phi(t)\} \) in terms of the transforms \( L\{f(t)\} \) and \( L\{k(t)\} \) of the given functions \( f \) and \( k \). The inverse transform of \( L\{\phi(t)\} \) is the solution of the original integral equation.