Problem 28

There are also equations, known as **integro-differential equations**, in which both derivatives and integrals of the unknown function appear. In each of Problems 26 through 28:

(a) Solve the given integro-differential equation by using the Laplace transform.

(b) By differentiating the integro-differential equation a sufficient number of times, convert it into an initial value problem.

(c) Solve the initial value problem in part (b), and verify that the solution is the same as the one in part (a).

\[
\phi'(t) + \phi(t) = \int_0^t \sin(t - \xi) \phi(\xi) \, d\xi, \quad \phi(0) = 1
\]