Problem 13

Transform Eqs. (2) for the parallel circuit into a single second order equation.

Solution

Eqs. (2) are on page 360 in the textbook.

\[
\frac{dI}{dt} = \frac{V}{L} \\
\frac{dV}{dt} = -\frac{I}{C} - \frac{V}{RC}
\]  

(2)

Solve this first equation for \( V \)

\[ V = L \frac{dI}{dt} \]

and then plug it into the second equation.

\[
\frac{d}{dt} \left( L \frac{dI}{dt} \right) = -\frac{I}{C} - \frac{1}{RC} \left( L \frac{dI}{dt} \right)
\]

\[ L \frac{d^2 I}{dt^2} = -\frac{I}{C} - \frac{L}{RC} \frac{dI}{dt} \]

Divide both sides by \( L \).

\[ \frac{d^2 I}{dt^2} = -\frac{I}{LC} - \frac{1}{RC} \frac{dI}{dt} \]

Bring all terms to the left side.

\[ \frac{d^2 I}{dt^2} + \frac{1}{RC} \frac{dI}{dt} + \frac{1}{LC} I = 0 \]