# Problem 17

Equations (1) can be derived by drawing a free-body diagram showing the forces acting on each mass. Figure 7.1.3*a* shows the situation when the displacements $x_1$ and $x_2$ of the two masses are both positive (to the right) and $x_2 > x_1$. Then springs 1 and 2 are elongated and spring 3 is compressed, giving rise to forces as shown in Figure 7.1.3*b*. Use Newton's law ($F = ma$) to derive Eqs. (1).

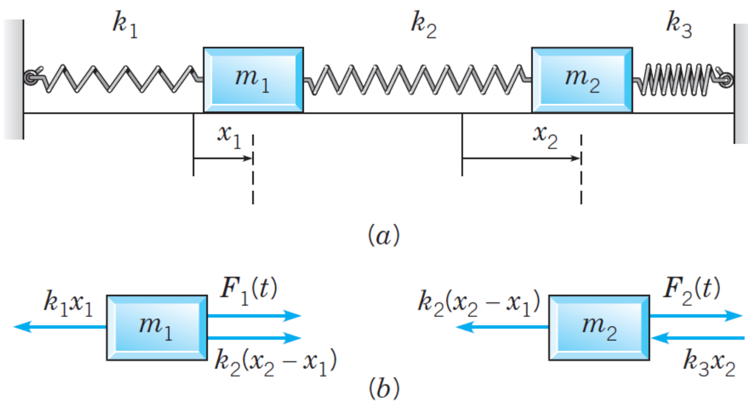


**FIGURE 7.1.3** (*a*) The displacements $x_1$ and $x_2$ are both positive. (*b*) The free-body diagram for the spring–mass system.

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### Solution

Newton's second law is the governing (vector) equation of motion.

$$\sum \mathbf{F} = m\mathbf{a}$$

Since the masses move in only one dimension, there's only one scalar equation for each mass.

$$\sum F_1 = m_1 x_1''$$
$$\sum F_2 = m_2 x_2''$$

Use the provided free-body diagrams to write each left side, noting that $x_1$ and $x_2$ are increasing to the right.

$$F_1(t) + k_2(x_2 - x_1) - k_1 x_1 = m_1 x_1''$$
$$F_2(t) - k_3 x_2 - k_2(x_2 - x_1) = m_2 x_2''$$

These are the equations in (1) on page 360 of the textbook.

$$\begin{aligned} m_1 \frac{d^2 x_1}{dt^2} &= -(k_1 + k_2) x_1 + k_2 x_2 + F_1(t) \\ m_2 \frac{d^2 x_2}{dt^2} &= k_2 x_1 - (k_2 + k_3) x_2 + F_2(t) \end{aligned} \tag{1}$$