Problem 18

Transform the system (1) into a system of first order equations by letting $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_1'$, and $y_4 = x_2'$.

Solution

The system in equation (1) is on page 360 of the textbook.

$$\begin{aligned} m_1\frac{d^2x_1}{dt^2} &= -(k_1+k_2)x_1 + k_2x_2 + F_1(t) \\ m_2\frac{d^2x_2}{dt^2} &= k_2x_1 - (k_2+k_3)x_2 + F_2(t) \end{aligned} \tag{1}$$

First, make the substitutions, $y_1 = x_1$ and $y_2 = x_2$.

$$m_1\frac{d^2y_1}{dt^2} = -(k_1+k_2)y_1 + k_2y_2 + F_1(t)$$

$$m_2\frac{d^2y_2}{dt^2} = k_2y_1 - (k_2+k_3)y_2 + F_2(t)$$

Secondly, make the substitutions, $y_3 = y_1'$ and $y_4 = y_2'$.

$$m_1\frac{dy_3}{dt} = -(k_1+k_2)y_1 + k_2y_2 + F_1(t)$$

$$m_2\frac{dy_4}{dt} = k_2y_1 - (k_2+k_3)y_2 + F_2(t)$$

As a result, the system of two second-order ODEs has become a system of four first-order ODEs.

$$\begin{aligned} y_1' &= y_3 \\ y_2' &= y_4 \\ m_1y_3' &= -(k_1+k_2)y_1 + k_2y_2 + F_1(t) \\ m_2y_4' &= k_2y_1 - (k_2+k_3)y_2 + F_2(t) \end{aligned}$$