Problem 4

In each of Problems 1 through 4, transform the given equation into a system of first order equations.

\[ u^{(4)} - u = 0 \]

Solution

Let \( u = x_1 \).

\[ x_1^{(4)} - x_1 = 0 \]

Let \( x_2 = x_1' \).

\[ x_2''' - x_1 = 0 \]

Let \( x_3 = x_2' \).

\[ x_3'' - x_1 = 0 \]

Finally, let \( x_4 = x_3' \).

\[ x_4' - x_1 = 0 \]

By making these substitutions, the original fourth-order ODE has become a system of first-order ODEs.

\[
\begin{align*}
  x_1' &= x_2 \\
  x_2' &= x_3 \\
  x_3' &= x_4 \\
  x_4' &= x_1
\end{align*}
\]