Problem 5

In each of Problems 5 and 6, transform the given initial value problem into an initial value problem for two first order equations.

\[ u'' + 0.25u' + 4u = 2 \cos 3t, \quad u(0) = 1, \quad u'(0) = -2 \]

Solution

Let \( u = x_1 \).

\[ x_1'' + 0.25x_1' + 4x_1 = 2 \cos 3t, \quad x_1(0) = 1, \quad x_1'(0) = -2 \]

Finally, let \( x_2 = x_1' \).

\[ x_2' + 0.25x_2 + 4x_1 = 2 \cos 3t, \quad x_1(0) = 1, \quad x_2(0) = -2 \]

By making these substitutions, the original initial value problem has become a system of first-order ODEs,

\[
\begin{cases}
  x_1' = x_2 \\
  x_2' = -4x_1 - 0.25x_2 + 2 \cos 3t
\end{cases}
\]

subject to the initial conditions,

\[ x_1(0) = 1 \quad \text{and} \quad x_2(0) = -2. \]