Problem 5

In each of Problems 5 and 6, transform the given initial value problem into an initial value problem for two first order equations.

$$u'' + 0.25u' + 4u = 2\cos 3t, \qquad u(0) = 1, \quad u'(0) = -2$$

Solution

Let $u = x_1$.

$$x_1'' + 0.25x_1' + 4x_1 = 2\cos 3t, \qquad x_1(0) = 1, \quad x_1'(0) = -2$$

Finally, let $x_2 = x_1'$.

$$x_2' + 0.25x_2 + 4x_1 = 2\cos 3t, \qquad x_1(0) = 1, \quad x_2(0) = -2$$

By making these substitutions, the original initial value problem has become a system of first-order ODEs,

$$\begin{cases} x_1' = x_2 \\ x_2' = -4x_1 - 0.25x_2 + 2\cos 3t \end{cases},$$

subject to the initial conditions,

$$x_1(0) = 1 \quad \text{and} \quad x_2(0) = -2.$$