

Problem 6

In each of Problems 5 and 6, transform the given initial value problem into an initial value problem for two first order equations.

$$u'' + p(t)u' + q(t)u = g(t), \quad u(0) = u_0, \quad u'(0) = u'_0$$

Solution

Let $u = x_1$.

$$x_1'' + p(t)x_1' + q(t)x_1 = g(t), \quad x_1(0) = u_0, \quad x_1'(0) = u'_0$$

Finally, let $x_2 = x_1'$.

$$x_2' + p(t)x_2 + q(t)x_1 = g(t), \quad x_1(0) = u_0, \quad x_2(0) = u'_0$$

By making these substitutions, the original initial value problem has become a system of first-order ODEs,

$$\begin{cases} x_1' = x_2 \\ x_2' = -q(t)x_1 - p(t)x_2 + g(t) \end{cases},$$

subject to the initial conditions,

$$x_1(0) = u_0 \quad \text{and} \quad x_2(0) = u'_0.$$