

Problem 1

If $\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix}$, find

(a) $2\mathbf{A} + \mathbf{B}$

(b) $\mathbf{A} - 4\mathbf{B}$

(c) \mathbf{AB}

(d) \mathbf{BA}

Solution

$$\begin{aligned} 2\mathbf{A} + \mathbf{B} &= 2 \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -4 & 0 \\ 6 & 4 & -2 \\ -4 & 2 & 6 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 6 & -6 & 3 \\ 5 & 9 & -2 \\ 2 & 3 & 8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{A} - 4\mathbf{B} &= \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{pmatrix} - 4 \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{pmatrix} - \begin{pmatrix} 16 & -8 & 12 \\ -4 & 20 & 0 \\ 24 & 4 & 8 \end{pmatrix} \\ &= \begin{pmatrix} -15 & 6 & -12 \\ 7 & -18 & -1 \\ -26 & -3 & -5 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} (1)(4) + (-2)(-1) + (0)(6) & (1)(-2) + (-2)(5) + (0)(1) & (1)(3) + (-2)(0) + (0)(2) \\ (3)(4) + (2)(-1) + (-1)(6) & (3)(-2) + (2)(5) + (-1)(1) & (3)(3) + (2)(0) + (-1)(2) \\ (-2)(4) + (1)(-1) + (3)(6) & (-2)(-2) + (1)(5) + (3)(1) & (-2)(3) + (1)(0) + (3)(2) \end{pmatrix} \\ &= \begin{pmatrix} 6 & -12 & 3 \\ 4 & 3 & 7 \\ 9 & 12 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}\mathbf{BA} &= \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} (4)(1) + (-2)(3) + (3)(-2) & (4)(-2) + (-2)(2) + (3)(1) & (4)(0) + (-2)(-1) + (3)(3) \\ (-1)(1) + (5)(3) + (0)(-2) & (-1)(-2) + (5)(2) + (0)(1) & (-1)(0) + (5)(-1) + (0)(3) \\ (6)(1) + (1)(3) + (2)(-2) & (6)(-2) + (1)(2) + (2)(1) & (6)(0) + (1)(-1) + (2)(3) \end{pmatrix} \\ &= \begin{pmatrix} -8 & -9 & 11 \\ 14 & 12 & -5 \\ 5 & -8 & 5 \end{pmatrix}\end{aligned}$$