Problem 10

In each of Problems 10 through 19, either compute the inverse of the given matrix, or else show that it is singular.

$$\begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$$

Solution

Start by calculating the determinant.

$$\det \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} = (1)(3) - (4)(-2) = 11$$

Since it's not zero, an inverse for the given matrix exists.

$$\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{array}\right)$$

The aim is to make the left side of the augmented matrix 1's and 0's as the right side is now. Since the top left entry is 1 already, we move on to the bottom left entry. To make it zero, multiply both sides of the first row by 2 and add it to the second row.

$$\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 11 & 2 & 1 \end{array}\right)$$

To make the bottom right entry 1, divide the bottom row by 11.

$$\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array}\right)$$

To make the top right entry 0, multiply the bottom row by -4 and add it to the first row.

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{11} & -\frac{4}{11} \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array}\right)$$

Therefore, the inverse matrix is

$$\begin{pmatrix} \frac{3}{11} & -\frac{4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{pmatrix}.$$