Problem 11

In each of Problems 10 through 19, either compute the inverse of the given matrix, or else show that it is singular.

$$\begin{pmatrix} 3 & -1 \\ 6 & 2 \end{pmatrix}$$

Solution

Start by calculating the determinant.

$$\det \begin{pmatrix} 3 & -1 \\ 6 & 2 \end{pmatrix} = (3)(2) - (-1)(6) = 12$$

Since it's not zero, an inverse for the given matrix exists.

$$\left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 6 & 2 & 0 & 1 \end{array}\right)$$

The aim is to make the left side of the augmented matrix 1's and 0's as the right side is now. Divide the bottom row by 2.

$$\left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 3 & 1 & 0 & \frac{1}{2} \end{array}\right)$$

To make the bottom left entry 0, multiply the top row by -1 and add it to the second row.

$$\left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 0 & 2 & -1 & \frac{1}{2} \end{array}\right)$$

Divide the bottom row by 2 to make the bottom right entry 1 again.

$$\left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{4} \end{array}\right)$$

Add the bottom row to the top row to make the top right entry 0.

$$\left(\begin{array}{cc|cc} 3 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{4} \end{array}\right)$$

Finally, divide the first row by 3 to make the top left entry 1.

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{6} & \frac{1}{12} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{4} \end{array}\right)$$

Therefore, the inverse matrix is

$$\begin{pmatrix} \frac{1}{6} & \frac{1}{12} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix}.$$