

Problem 12

In each of Problems 10 through 19, either compute the inverse of the given matrix, or else show that it is singular.

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

Solution

Start by calculating the determinant.

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} = 1 \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2 \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3 \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = 1(24 - 25) - 2(12 - 15) + 3(10 - 12) = -1$$

Since it's not zero, an inverse for the given matrix exists.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array} \right)$$

The aim is to make the left side 1's and 0's as the right side is now. The top left entry is 1 already. Make the mid-left entry 0 by multiplying the first row by -2 and adding it to the second row.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \end{array} \right)$$

Switch row 2 with row 3.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 5 & 6 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array} \right)$$

Multiply the third row by 6 and add it to the second row to make the mid-right entry 0.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 5 & 0 & -12 & 6 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array} \right)$$

Multiply the third row by 3 and add it to the first row to make the top right entry 0.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 3 & 0 \\ 3 & 5 & 0 & -12 & 6 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array} \right)$$

Multiply the third row by -1 to make the bottom right entry 1.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 3 & 0 \\ 3 & 5 & 0 & -12 & 6 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right)$$

Multiply the first row by -3 and add it to the second row to make the mid-left entry 0.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 3 & 0 \\ 0 & -1 & 0 & 3 & -3 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right)$$

Multiply the second row by 2 and add it to the first row to make the top middle entry 0.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & -1 & 0 & 3 & -3 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right)$$

Multiply the second row by -1 to make the middle entry 1.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 2 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right)$$

Therefore, the inverse of the given matrix is

$$\begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$$