

Problem 22

In each of Problems 22 through 24, verify that the given vector satisfies the given differential equation.

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} e^{2t}$$

Solution

Check to see that the given matrix satisfies the ODE.

$$\begin{aligned} \begin{pmatrix} 4e^{2t} \\ 2e^{2t} \end{pmatrix}' &\stackrel{?}{=} \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} 4e^{2t} \\ 2e^{2t} \end{pmatrix} \\ \begin{pmatrix} 4(2)e^{2t} \\ 2(2)e^{2t} \end{pmatrix} &\stackrel{?}{=} \begin{pmatrix} 12e^{2t} - 4e^{2t} \\ 8e^{2t} - 4e^{2t} \end{pmatrix} \\ \begin{pmatrix} 8e^{2t} \\ 4e^{2t} \end{pmatrix} &= \begin{pmatrix} 8e^{2t} \\ 4e^{2t} \end{pmatrix} \end{aligned}$$

Because the left and right sides are the same, the given matrix for \mathbf{x} is indeed a solution.