

Problem 23

In each of Problems 22 through 24, verify that the given vector satisfies the given differential equation.

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t, \quad \mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^t$$

Solution

Rewrite the given matrix.

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^t = \begin{pmatrix} e^t \\ 0 \end{pmatrix} + \begin{pmatrix} 2te^t \\ 2te^t \end{pmatrix} = \begin{pmatrix} e^t + 2te^t \\ 2te^t \end{pmatrix}$$

Check to see that it satisfies the ODE.

$$\begin{aligned} \begin{pmatrix} e^t + 2te^t \\ 2te^t \end{pmatrix}' &\stackrel{?}{=} \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} e^t + 2te^t \\ 2te^t \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t \\ \begin{pmatrix} e^t + 2e^t + 2te^t \\ 2e^t + 2te^t \end{pmatrix} &\stackrel{?}{=} \begin{pmatrix} 2(e^t + 2te^t) - 2te^t \\ 3(e^t + 2te^t) - 4te^t \end{pmatrix} + \begin{pmatrix} e^t \\ -e^t \end{pmatrix} \\ \begin{pmatrix} 3e^t + 2te^t \\ 2e^t + 2te^t \end{pmatrix} &\stackrel{?}{=} \begin{pmatrix} 2e^t + 2te^t \\ 3e^t + 2te^t \end{pmatrix} + \begin{pmatrix} e^t \\ -e^t \end{pmatrix} \\ \begin{pmatrix} 3e^t + 2te^t \\ 2e^t + 2te^t \end{pmatrix} &= \begin{pmatrix} 3e^t + 2te^t \\ 2e^t + 2te^t \end{pmatrix} \end{aligned}$$

Because the left and right sides are the same, the given matrix for \mathbf{x} is indeed a solution.