Problem 24

In each of Problems 22 through 24, verify that the given vector satisfies the given differential equation.

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x} = \begin{pmatrix} 6 \\ -8 \\ -4 \end{pmatrix} e^{-t} + 2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}$$

Solution

Rewrite the given matrix.

$$\mathbf{x} = \begin{pmatrix} 6 \\ -8 \\ -4 \end{pmatrix} e^{-t} + 2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} = \begin{pmatrix} 6e^{-t} \\ -8e^{-t} \\ -4e^{-t} \end{pmatrix} + \begin{pmatrix} 0 \\ 2e^{2t} \\ -2e^{2t} \end{pmatrix} = \begin{pmatrix} 6e^{-t} \\ -8e^{-t} + 2e^{2t} \\ -4e^{-t} - 2e^{2t} \end{pmatrix}$$

Check to see that it satisfies the ODE.

$$\begin{pmatrix} 6e^{-t} \\ -8e^{-t} + 2e^{2t} \\ -4e^{-t} - 2e^{2t} \end{pmatrix}' \stackrel{?}{=} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 6e^{-t} \\ -8e^{-t} + 2e^{2t} \\ -4e^{-t} - 2e^{2t} \end{pmatrix}$$

$$\begin{pmatrix} 6(-1)e^{-t} \\ -8(-1)e^{-t} + 2(2)e^{2t} \\ -4(-1)e^{-t} - 2(2)e^{2t} \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} 6e^{-t} - 8e^{-t} + 2e^{2t} - 4e^{-t} - 2e^{2t} \\ 12e^{-t} - 8e^{-t} + 2e^{2t} + 4e^{-t} + 2e^{2t} \\ 8e^{-t} - 2e^{2t} - 4e^{-t} - 2e^{2t} \end{pmatrix}$$

$$\begin{pmatrix} -6e^{-t} \\ 8e^{-t} + 4e^{2t} \\ 4e^{-t} - 4e^{2t} \end{pmatrix} = \begin{pmatrix} -6e^{-t} \\ 8e^{-t} + 4e^{2t} \\ 4e^{-t} - 4e^{2t} \end{pmatrix}$$

Because the left and right sides are the same, the given matrix for $\mathbf{\Psi}(t)$ is indeed a solution.