

Problem 26

In each of Problems 25 and 26, verify that the given matrix satisfies the given differential equation.

$$\Psi' = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \Psi, \quad \Psi(t) = \begin{pmatrix} e^t & e^{-2t} & e^{3t} \\ -4e^t & -e^{-2t} & 2e^{3t} \\ -e^t & -e^{-2t} & e^{3t} \end{pmatrix}$$

Solution

Check to see that the given matrix satisfies the ODE.

$$\begin{aligned} \begin{pmatrix} e^t & e^{-2t} & e^{3t} \\ -4e^t & -e^{-2t} & 2e^{3t} \\ -e^t & -e^{-2t} & e^{3t} \end{pmatrix}' &\stackrel{?}{=} \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} e^t & e^{-2t} & e^{3t} \\ -4e^t & -e^{-2t} & 2e^{3t} \\ -e^t & -e^{-2t} & e^{3t} \end{pmatrix} \\ \begin{pmatrix} e^t & (-2)e^{-2t} & (3)e^{3t} \\ -4e^t & -(-2)e^{-2t} & 2(3)e^{3t} \\ -e^t & -(-2)e^{-2t} & (3)e^{3t} \end{pmatrix} &\stackrel{?}{=} \begin{pmatrix} e^t + 4e^t - 4e^t & e^{-2t} + e^{-2t} - 4e^{-2t} & e^{3t} - 2e^{3t} + 4e^{3t} \\ 3e^t - 8e^t + e^t & 3e^{-2t} - 2e^{-2t} + e^{-2t} & 3e^{3t} + 4e^{3t} - e^{3t} \\ 2e^t - 4e^t + e^t & 2e^{-2t} - e^{-2t} + e^{-2t} & 2e^{3t} + 2e^{3t} - e^{3t} \end{pmatrix} \\ \begin{pmatrix} e^t & -2e^{-2t} & 3e^{3t} \\ -4e^t & 2e^{-2t} & 6e^{3t} \\ -e^t & 2e^{-2t} & 3e^{3t} \end{pmatrix} &= \begin{pmatrix} e^t & -2e^{-2t} & 3e^{3t} \\ -4e^t & 2e^{-2t} & 6e^{3t} \\ -e^t & 2e^{-2t} & 3e^{3t} \end{pmatrix} \end{aligned}$$

Because the left and right sides are the same, the given matrix for $\Psi(t)$ is indeed a solution.