Problem 3

If $\mathbf{A} = \begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & -1 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -1 & -1 \\ -2 & 1 & 0 \end{pmatrix}$, find

(a) $\mathbf{A}^T$

(b) $\mathbf{B}^T$

(c) $\mathbf{A}^T + \mathbf{B}^T$

(d) $(\mathbf{A} + \mathbf{B})^T$

Solution

To obtain the transpose of a matrix, switch the elements about the a_{11}-a_{33} long diagonal.

$$\mathbf{A}^T = \begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -3 & 1 \end{pmatrix}$$

$$\mathbf{B}^T = \begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & -1 & 0 \end{pmatrix}$$

$$\begin{aligned} \mathbf{A}^T + \mathbf{B}^T &= \begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 4 & 0 \\ 3 & -1 & 0 \\ 5 & -4 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{A} + \mathbf{B} &= \begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 3 & -1 & -1 \\ -2 & 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 3 & 5 \\ 4 & -1 & -4 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

$$(\mathbf{A} + \mathbf{B})^T = \begin{pmatrix} -1 & 4 & 0 \\ 3 & -1 & 0 \\ 5 & -4 & 1 \end{pmatrix}$$