

Problem 8

If $\mathbf{x} = \begin{pmatrix} 2 \\ 3i \\ 1-i \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} -1+i \\ 2 \\ 3-i \end{pmatrix}$, find

(a) $\mathbf{x}^T \mathbf{y}$

(b) $\mathbf{y}^T \mathbf{y}$

(c) (\mathbf{x}, \mathbf{y})

(d) (\mathbf{y}, \mathbf{y})

Solution

$$\mathbf{x}^T \mathbf{y} = (2 \ 3i \ 1-i) \begin{pmatrix} -1+i \\ 2 \\ 3-i \end{pmatrix} = (2)(-1+i) + (3i)(2) + (1-i)(3-i) = 4i$$

$$\mathbf{y}^T \mathbf{y} = (-1+i \ 2 \ 3-i) \begin{pmatrix} -1+i \\ 2 \\ 3-i \end{pmatrix} = (-1+i)^2 + (2)^2 + (3-i)^2 = 12 - 8i$$

$$(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \bar{\mathbf{y}} = (2 \ 3i \ 1-i) \begin{pmatrix} -1-i \\ 2 \\ 3+i \end{pmatrix} = (2)(-1-i) + (3i)(2) + (1-i)(3+i) = 2 + 2i$$

$$(\mathbf{y}, \mathbf{y}) = \mathbf{y}^T \bar{\mathbf{y}} = (-1+i \ 2 \ 3-i) \begin{pmatrix} -1-i \\ 2 \\ 3+i \end{pmatrix} = (-1+i)(-1-i) + (2)(2) + (3-i)(3+i) = 16$$