Problem 9

If $\mathbf{x} = \begin{pmatrix} 1-2i \\ i \\ 2 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 2 \\ 3-i \\ 1+2i \end{pmatrix}$, show that

(a) $\mathbf{x}^T\mathbf{y} = \mathbf{y}^T\mathbf{x}$ (b) $(\mathbf{x}, \mathbf{y}) = \overline{(\mathbf{y}, \mathbf{x})}$

Solution

$$\mathbf{x}^T\mathbf{y} = \begin{pmatrix} 1-2i & i & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3-i \\ 1+2i \end{pmatrix} = (1-2i)(2) + (i)(3-i) + (2)(1+2i) = 5+3i$$

$$\mathbf{y}^T\mathbf{x} = \begin{pmatrix} 2 & 3-i & 1+2i \end{pmatrix} \begin{pmatrix} 1-2i \\ i \\ 2 \end{pmatrix} = (2)(1-2i) + (3-i)(i) + (1+2i)(2) = 5+3i$$

Therefore, $\mathbf{x}^T\mathbf{y} = \mathbf{y}^T\mathbf{x}$.

$$(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T\bar{\mathbf{y}} = \begin{pmatrix} 1-2i & i & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3+i \\ 1-2i \end{pmatrix} = (1-2i)(2) + (i)(3+i) + (2)(1-2i) = 3-5i$$

$$(\mathbf{y}, \mathbf{x}) = \mathbf{y}^T\bar{\mathbf{x}} = \begin{pmatrix} 2 & 3-i & 1+2i \end{pmatrix} \begin{pmatrix} 1+2i \\ -i \\ 2 \end{pmatrix} = (2)(1+2i) + (3-i)(-i) + (1+2i)(2) = 3+5i$$

Therefore, $(\mathbf{x}, \mathbf{y}) = \overline{(\mathbf{y}, \mathbf{x})}$.