

Problem 5

In each of Problems 1 through 6, either solve the given system of equations, or else show that there is no solution.

$$\begin{aligned}x_1 - x_3 &= 0 \\3x_1 + x_2 + x_3 &= 0 \\-x_1 + x_2 + 2x_3 &= 0\end{aligned}$$

Solution

Start by calculating the determinant of the coefficient matrix.

$$\det \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix} = 1 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 1(2 - 1) - 1(3 + 1) = -3$$

Since it's not zero, a unique solution to the system of equations exists. Solve the first equation for x_3 .

$$x_3 = x_1$$

Substitute this result into the other two equations.

$$\begin{aligned}3x_1 + x_2 + (x_1) &= 0 \\-x_1 + x_2 + 2(x_1) &= 0\end{aligned}$$

Simplify the left sides.

$$\begin{aligned}4x_1 + x_2 &= 0 \\x_1 + x_2 &= 0\end{aligned}$$

Solve the second equation for x_1 ,

$$x_1 = -x_2,$$

and substitute it into the first equation.

$$4(-x_2) + x_2 = 0$$

Solve for x_2 .

$$x_2 = 0$$

That means

$$x_3 = x_1 = -(0) = 0.$$

Therefore,

$$\{0, 0, 0\}.$$