

Problem 7

In each of Problems 7 through 11, determine whether the members of the given set of vectors are linearly independent. If they are linearly dependent, find a linear relation among them. The vectors are written as row vectors to save space but may be considered as column vectors; that is, the transposes of the given vectors may be used instead of the vectors themselves.

$$\mathbf{x}^{(1)} = (1, 1, 0), \quad \mathbf{x}^{(2)} = (0, 1, 1), \quad \mathbf{x}^{(3)} = (1, 0, 1)$$

Solution

The three vectors are linearly dependent if there exists a nontrivial solution to

$$c_1\mathbf{x}^{(1)} + c_2\mathbf{x}^{(2)} + c_3\mathbf{x}^{(3)} = \mathbf{0}$$

for c_1 , c_2 , and c_3 . Rewrite this equation.

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Calculate the determinant of the coefficient matrix.

$$\det \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = 1 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1(1 - 0) + 1(1 - 0) = 2$$

Since it's not zero, there is a unique solution for c_1 , c_2 , and c_3 .

$$c_1 + c_3 = 0$$

$$c_1 + c_2 = 0$$

$$c_2 + c_3 = 0$$

By inspection, $c_1 = c_2 = c_3 = 0$ is the (trivial) solution. Therefore, the three given vectors are linearly independent.