Problem 19

The system $tx' = Ax$ is analogous to the second order Euler equation (Section 5.4). Assuming that $x = \xi t^r$, where $\xi$ is a constant vector, show that $\xi$ and $r$ must satisfy $(A - rI)\xi = 0$ in order to obtain nontrivial solutions of the given differential equation.

Solution

Because this looks like an Euler equation, make the substitution

$$x = \xi t^r,$$

where $\xi$ is a constant.

$$t(\xi t^{-1}) = A\xi t^r$$

$$rt^r \xi = A\xi t^r$$

Divide both sides by $t^r$.

$$r\xi = A\xi$$

Bring both terms to the right side.

$$0 = A\xi - r\xi$$

$$= A\xi - rI\xi$$

$$= (A - rI)\xi$$

The system of ODEs has therefore become an eigenvalue problem.