Exercise 1.29

What is the kinetic energy and velocity of the aluminum sphere in Problem 1.4 at the moment it hits the ground? (Assume that energy is conserved during the fall and that 100% of the sphere's initial potential energy is converted to kinetic energy by the time impact occurs.)

Solution

The work required to raise the aluminum sphere off the floor a distance d is

$$w = F \times d.$$

According to Newton's second law, $F = ma$.

$$w = (ma) \times d.$$

Since 100% of the sphere's potential energy is converted to kinetic energy, $w = \mathrm{KE} = (1/2)mv^2$.

$$(ma) \times d = \frac{1}{2}mv^2.$$

Now solve this equation for v.

$$mad = \frac{1}{2}mv^2$$

$$2mad = mv^2$$

$$2ad = v^2$$

$$v = \sqrt{2ad}$$

The acceleration due to gravity is $a = 9.8\ \mathrm{m/s^2}$, and from Exercise 1.4, $d = 2.2$ m.

$$v = \sqrt{2\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(2.2\ \mathrm{m})} \approx 6.6\ \frac{\mathrm{m}}{\mathrm{s}}$$

The kinetic energy the aluminum sphere has right before it hits the floor is

$$\mathrm{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\left[196\ \cancel{\mathrm{cm}^3} \times 2.70\ \frac{\cancel{\mathrm{g}}}{\cancel{\mathrm{cm}^3}} \times \frac{1\ \mathrm{kg}}{1000\ \cancel{\mathrm{g}}}\right]\left[2\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(2.2\ \mathrm{m})\right] \approx 11\ \mathrm{J}.$$

The mass of aluminum is obtained by multiplying the density and volume together—both are given in Exercise 1.4.