Exercise 3

Show that $(1+z)^2 = 1 + 2z + z^2$.

Solution

In order to show that these functions are equal, we have to show that the real and imaginary parts of both are the same. Substitute $z = x + iy$ and simplify both sides.

$$\begin{aligned} (1+z)^2 &= (1+x+iy)^2 \\ &= (1+x)^2 + 2(1+x)iy + i^2y^2 \\ &= (1+x)^2 - y^2 + 2iy(1+x) \end{aligned}$$

The real and imaginary parts of $(1+z)^2$ are $(1+x)^2 - y^2$ and $2y(1+x)$, respectively.

$$\begin{aligned} 1+2z+z^2 &= 1 + 2(x+iy) + (x+iy)^2 \\ &= 1 + 2x + 2iy + x^2 + 2ixy + i^2y^2 \\ &= 1 + 2x + x^2 - y^2 + 2iy(1+x) \\ &= (1+x)^2 - y^2 + 2iy(1+x) \end{aligned}$$

The real and imaginary parts of $1 + 2z + z^2$ are $(1+x)^2 - y^2$ and $2y(1+x)$, respectively. Therefore,

$$(1+z)^2 = 1 + 2z + z^2.$$