Exercise 5

Prove that multiplication of complex numbers is commutative, as stated at the beginning of Sec. 2.

Solution

Here we have to show that

\[ z_1 z_2 = z_2 z_1, \]

where \( z_1 \) and \( z_2 \) are complex numbers. Let \( z_1 = (x_1, y_1) \) and \( z_2 = (x_2, y_2) \) and assume that \( x_1, x_2, y_1, \) and \( y_2 \) are real numbers.

\[
\begin{align*}
  z_1 z_2 &= (x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, y_1 x_2 + x_1 y_2) \\
  z_2 z_1 &= (x_2, y_2)(x_1, y_1) = (x_2 x_1 - y_2 y_1, y_2 x_1 + x_2 y_1)
\end{align*}
\]

Because \( x_1 x_2 - y_1 y_2 = x_2 x_1 - y_2 y_1 \) and \( y_1 x_2 + x_1 y_2 = y_2 x_1 + x_2 y_1 \), the real and imaginary components of \( z_1 z_2 \) and \( z_2 z_1 \) are the same. Therefore,

\[ z_1 z_2 = z_2 z_1. \]