Exercise 6

Verify

(a) the associative law for addition of complex numbers, stated at the beginning of Sec. 2;

(b) the distributive law (3), Sec. 2.

Solution

Part (a)

The associative law for addition of complex numbers states that

\[(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3).\]

To verify it, let \(z_1 = 6i\) and \(z_2 = -1 + 2i\) and \(z_3 = 2 - 4i\). Check to see whether both sides are equal.

\[
[(6i) + (-1 + 2i)] + (2 - 4i) \overset{?}{=} (6i) + [(-1 + 2i) + (2 - 4i)]
\]

\[
[-1 + 8i] + 2 - 4i \overset{?}{=} 6i + [1 - 2i]
\]

\[
1 + 4i = 1 + 4i
\]

The associative law for addition is verified.

Part (b)

The distributive law states that

\[z(z_1 + z_2) = zz_1 + zz_2.\]

To verify it, let \(z_1\) and \(z_2\) be the same values as before and let \(z = 3 + 5i\). Check to see whether both sides are equal.

\[
(3 + 5i)[(6i) + (-1 + 2i)] \overset{?}{=} (3 + 5i)(6i) + (3 + 5i)(-1 + 2i)
\]

\[
(3 + 5i)[-1 + 8i] \overset{?}{=} (18i + 30i^2) + (-3 - 6i - 5i + 10i^2)
\]

\[
-3 + 24i - 5i + 40i^2 \overset{?}{=} 18i - 30 - 3 + i - 10
\]

\[
-3 + 19i - 40 \overset{?}{=} 19i - 43
\]

\[-43 + 19i = -43 + 19i
\]

The distributive law is verified.