Exercise 8

(a) Write $(x, y) + (u, v) = (x, y)$ and point out how it follows that the complex number $0 = (0, 0)$ is unique as an additive identity.

(b) Likewise, write $(x, y)(u, v) = (x, y)$ and show that the number $1 = (1, 0)$ is a unique multiplicative identity.

Solution

Part (a)

$$(x, y) + (u, v) = (x, y)$$

Adding the two complex numbers on the left side, we get

$$(x + u, y + v) = (x, y).$$

In order for equality to hold, the real and imaginary parts of both sides must be equal. That is,

$$x + u = x \quad \text{and} \quad y + v = y.$$

This is a system of two equations for two unknowns, u and v. Solving it yields $u = 0$ and $v = 0$. Therefore, $(0, 0) = 0 + 0i$ is the unique additive identity for complex numbers.

Part (b)

$$(x, y)(u, v) = (x, y)$$

Multiplying the two complex numbers on the left side, we get

$$(xu - yv, uy + xv) = (x, y).$$

In order for equality to hold, the real and imaginary parts of both sides must be equal. That is,

$$xu - yv = x \quad \text{and} \quad uy + xv = y.$$

This is a system of two equations for two unknowns, u and v. Solving it yields $u = 1$ and $v = 0$. Therefore, $(1, 0) = 1 + 0i$ is the unique multiplicative identity for complex numbers.