Exercise 8

(a) Write \((x, y) + (u, v) = (x, y)\) and point out how it follows that the complex number \(0 = (0, 0)\) is unique as an additive identity.

(b) Likewise, write \((x, y)(u, v) = (x, y)\) and show that the number \(1 = (1, 0)\) is a unique multiplicative identity.

Solution

Part (a)

\[(x, y) + (u, v) = (x, y)\]

Adding the two complex numbers on the left side, we get

\[(x + u, y + v) = (x, y)\].

In order for equality to hold, the real and imaginary parts of both sides must be equal. That is,

\[x + u = x\] and \[y + v = y\].

This is a system of two equations for two unknowns, \(u\) and \(v\). Solving it yields \(u = 0\) and \(v = 0\). Therefore, \((0, 0) = 0 + 0i\) is the unique additive identity for complex numbers.

Part (b)

\[(x, y)(u, v) = (x, y)\]

Multiplying the two complex numbers on the left side, we get

\[(xu - yv, uy + xv) = (x, y)\].

In order for equality to hold, the real and imaginary parts of both sides must be equal. That is,

\[xu - yv = x\] and \[uy + xv = y\].

This is a system of two equations for two unknowns, \(u\) and \(v\). Solving it yields \(u = 1\) and \(v = 0\). Therefore, \((1, 0) = 1 + 0i\) is the unique multiplicative identity for complex numbers.