

## Exercise 8

Use mathematical induction to verify the binomial formula (13) in Sec. 3. More precisely, note that the formula is true when  $n = 1$ . Then, assuming that it is valid when  $n = m$  where  $m$  denotes any positive integer, show that it must hold when  $n = m + 1$ .

*Suggestion:* When  $n = m + 1$ , write

$$\begin{aligned}(z_1 + z_2)^{m+1} &= (z_1 + z_2)(z_1 + z_2)^m = (z_2 + z_1) \sum_{k=0}^m \binom{m}{k} z_1^k z_2^{m-k} \\ &= \sum_{k=0}^m \binom{m}{k} z_1^k z_2^{m+1-k} + \sum_{k=0}^m \binom{m}{k} z_1^{k+1} z_2^{m-k}\end{aligned}$$

and replace  $k$  by  $k - 1$  in the last sum here to obtain

$$(z_1 + z_2)^{m+1} = z_2^{m+1} + \sum_{k=1}^m \left[ \binom{m}{k} + \binom{m}{k-1} \right] z_1^k z_2^{m+1-k} + z_1^{m+1}.$$

Finally, show how the right-hand side here becomes

$$z_2^{m+1} + \sum_{k=1}^m \binom{m+1}{k} z_1^k z_2^{m+1-k} + z_1^{m+1} = \sum_{k=0}^{m+1} \binom{m+1}{k} z_1^k z_2^{m+1-k}.$$