

Exercise 2

Verify inequalities (3), Sec. 4, involving $\operatorname{Re} z$, $\operatorname{Im} z$, and $|z|$.

Solution

Inequalities (3) in Sec. 4 are

$$\operatorname{Re} z \leq |\operatorname{Re} z| \leq |z| \quad \text{and} \quad \operatorname{Im} z \leq |\operatorname{Im} z| \leq |z|. \quad (3)$$

Suppose $z = x + iy$. Then the first inequality becomes

$$x \leq |x| \leq \sqrt{x^2 + y^2},$$

which is true. The second inequality becomes

$$y \leq |y| \leq \sqrt{x^2 + y^2},$$

which is also true.