Exercise 11
Use mathematical induction to show that when \( n = 2, 3, \ldots \),

\[
(a) \quad \overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}; \\
(b) \quad \overline{z_1 z_2 \cdots z_n} = \overline{z_1 z_2 \cdots z_n}.
\]

Solution
Part (a)
Start by showing that the result holds in the base case \( n = 2 \).

\[
\overline{z_1 + z_2} = \overline{\overline{z_1} + \overline{z_2}}
\]

This is property (2) in the text, which has been shown to be true. Now assume the inductive hypothesis,

\[
\overline{z_1 + z_2 + \cdots + z_k} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_k},
\]

and show that

\[
\overline{z_1 + z_2 + \cdots + z_k + z_{k+1}} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_k} + \overline{z_{k+1}}.
\]

Do so by grouping the first \( k \) terms, using the base case, and then using the inductive hypothesis.

\[
\overline{z_1 + z_2 + \cdots + z_k + z_{k+1}} = (\overline{z_1 + z_2 + \cdots + z_k}) + \overline{z_{k+1}}
\]

\[
= \overline{z_1} + \overline{z_2} + \cdots + \overline{z_k} + \overline{z_{k+1}}
\]

Therefore, by mathematical induction,

\[
\overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}.
\]

Part (b)
Start by showing that the result holds in the base case \( n = 2 \).

\[
\overline{z_1 z_2} = \overline{\overline{z_1} \overline{z_2}}
\]

This is property (4) in the text, which has been shown to be true. Now assume the inductive hypothesis,

\[
\overline{z_1 z_2 \cdots z_k} = \overline{\overline{z_1} \overline{z_2} \cdots \overline{z_k}}
\]

and show that

\[
\overline{z_1 z_2 \cdots z_k z_{k+1}} = \overline{\overline{z_1} \overline{z_2} \cdots \overline{z_k} \overline{z_{k+1}}}
\]

Do so by grouping the first \( k \) terms, using the base case, and then using the inductive hypothesis.

\[
\overline{z_1 z_2 \cdots z_k z_{k+1}} = (\overline{z_1 z_2 \cdots z_k}) z_{k+1}
\]

\[
= \overline{z_1 z_2 \cdots z_k} \overline{z_{k+1}}
\]

Therefore, by mathematical induction,

\[
\overline{z_1 z_2 \cdots z_n} = \overline{\overline{z_1} \overline{z_2} \cdots \overline{z_n}}
\]

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