Exercise 12

Let a_0, a_1, a_2, …, a_n ($n \geq 1$) denote *real* numbers, and let z be any complex number. With the aid of the results in Exercise 11, show that

$$\overline{a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n} = a_0 + a_1 \bar{z} + a_2 \bar{z}^2 + \cdots + a_n \bar{z}^n.$$

Solution

$$\begin{aligned}
\overline{a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n} &= \overline{a_0} + \overline{a_1 z} + \overline{a_2 z^2} + \cdots + \overline{a_n z^n} \\
&= \overline{a_0} + \overline{a_1}\bar{z} + \overline{a_2}\overline{z^2} + \cdots + \overline{a_n}\overline{z^n} \\
&= a_0 + a_1 \bar{z} + a_2 \overline{z^2} + \cdots + a_n \overline{z^n} \\
&= a_0 + a_1 \bar{z} + a_2 \bar{z}^2 + \cdots + a_n \bar{z}^n
\end{aligned}$$