Exercise 15

Follow the steps below to give an algebraic derivation of the triangle inequality (Sec. 4)

(a) Show that

$$|z_1 + z_2|^2 = (z_1 + z_2)(\overline{z_1} + \overline{z_2}) = z_1\overline{z_1} + (z_1\overline{z_2} + z_2\overline{z_1}) + z_2\overline{z_2}.$$  

(b) Point out why

$$z_1\overline{z_2} + z_2\overline{z_1} = 2 \text{Re}(z_1\overline{z_2}) \leq 2|z_1||z_2|.$$  

(c) Use the results in parts (a) and (b) to obtain the inequality

$$|z_1 + z_2|^2 \leq (|z_1| + |z_2|)^2,$$

and note how the triangle inequality follows.