

Exercise 6

Use results in Sec. 5 to show that when z_2 and z_3 are nonzero,

$$(a) \overline{\left(\frac{z_1}{z_2 z_3}\right)} = \frac{\bar{z}_1}{\bar{z}_2 \bar{z}_3}; \quad (b) \left|\frac{z_1}{z_2 z_3}\right| = \frac{|z_1|}{|z_2||z_3|}.$$

Solution**Part (a)**

$$\begin{aligned} \overline{\left(\frac{z_1}{z_2 z_3}\right)} &= \overline{\left(\frac{z_1}{(z_2 z_3)}\right)} \\ &= \frac{\bar{z}_1}{\bar{z}_2 \bar{z}_3} \\ &= \frac{\bar{z}_1}{\bar{z}_2 \bar{z}_3} \end{aligned}$$

Part (b)

$$\begin{aligned} \left|\frac{z_1}{z_2 z_3}\right|^2 &= \left(\frac{z_1}{z_2 z_3}\right) \overline{\left(\frac{z_1}{z_2 z_3}\right)} \\ &= \left(\frac{z_1}{z_2 z_3}\right) \left(\frac{\bar{z}_1}{\bar{z}_2 \bar{z}_3}\right) \\ &= \frac{z_1 \bar{z}_1}{z_2 z_3 \bar{z}_2 \bar{z}_3} \\ &= \frac{|z_1|^2}{z_2 \bar{z}_2 z_3 \bar{z}_3} \\ &= \frac{|z_1|^2}{|z_2|^2 |z_3|^2} \\ &= \left(\frac{|z_1|}{|z_2||z_3|}\right)^2 \end{aligned}$$

Therefore,

$$\left|\frac{z_1}{z_2 z_3}\right| = \frac{|z_1|}{|z_2||z_3|}$$

for any three complex numbers, z_1 and z_2 and z_3 , with z_2 and z_3 being nonzero.