

Exercise 7

Show that

$$|\operatorname{Re}(2 + \bar{z} + z^3)| \leq 4 \quad \text{when } |z| \leq 1.$$

Solution

$$\begin{aligned} |\operatorname{Re}(2 + \bar{z} + z^3)| &\leq |2 + \bar{z} + z^3| \\ &\leq |2| + |\bar{z}| + |z^3| \\ &= 2 + |z| + |z|^3 \\ &\leq 2 + 1 + 1 \\ &= 4 \end{aligned}$$

Therefore,

$$|\operatorname{Re}(2 + \bar{z} + z^3)| \leq 4 \quad \text{when } |z| \leq 1.$$