Exercise 2

Show that $(a)\ |e^{i\theta}| = 1;\quad (b)\ \overline{e^{i\theta}} = e^{-i\theta}$.

Solution

Part (a)

Use Euler's formula

$$\begin{aligned}
|e^{i\theta}|^2 &= e^{i\theta}\overline{e^{i\theta}} \\
&= (\cos\theta + i\sin\theta)(\overline{\cos\theta + i\sin\theta}) \\
&= (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta) \\
&= \cos^2\theta - i^2\sin^2\theta \\
&= \cos^2\theta + \sin^2\theta \\
&= 1
\end{aligned}$$

Therefore, taking the square root of both sides,

$$|e^{i\theta}| = 1.$$

Part (b)

Use Euler's formula.

$$\begin{aligned}
\overline{e^{i\theta}} &= \overline{\cos\theta + i\sin\theta} \\
&= \overline{\cos\theta} + \overline{i\sin\theta} \\
&= \cos\theta + (-i)\sin\theta \\
&= \cos(-\theta) + i\sin(-\theta) \\
&= e^{i(-\theta)} \\
&= e^{-i\theta}
\end{aligned}$$