

Exercise 8

Prove that two nonzero complex numbers z_1 and z_2 have the same moduli if and only if there are complex numbers c_1 and c_2 such that $z_1 = c_1 c_2$ and $z_2 = c_1 \bar{c}_2$.

Suggestion: Note that

$$\exp\left(i\frac{\theta_1 + \theta_2}{2}\right) \exp\left(i\frac{\theta_1 - \theta_2}{2}\right) = \exp(i\theta_1)$$

and

$$\exp\left(i\frac{\theta_1 + \theta_2}{2}\right) \overline{\exp\left(i\frac{\theta_1 - \theta_2}{2}\right)} = \exp(i\theta_2).$$