Exercise 8

Prove that two nonzero complex numbers $z_1$ and $z_2$ have the same moduli if and only if there are complex numbers $c_1$ and $c_2$ such that $z_1 = c_1c_2$ and $z_2 = c_1\overline{c_2}$.

*Suggestions:* Note that

$$\exp\left(i\frac{\theta_1 + \theta_2}{2}\right)\exp\left(i\frac{\theta_1 - \theta_2}{2}\right) = \exp(i\theta_1)$$

and

$$\exp\left(i\frac{\theta_1 + \theta_2}{2}\right)\exp\left(i\frac{\theta_1 - \theta_2}{2}\right) = \exp(i\theta_2).$$