

Exercise 4

In Exercises 1 through 4, take the indented contour in Fig. 101 (Sec. 82).

Use the function

$$f(z) = \frac{(\log z)^2}{z^2 + 1} \quad \left(|z| > 0, -\frac{\pi}{2} < \arg z < \frac{3\pi}{2} \right)$$

to show that

$$\int_0^\infty \frac{(\ln x)^2}{x^2 + 1} dx = \frac{\pi^3}{8}, \quad \int_0^\infty \frac{\ln x}{x^2 + 1} dx = 0.$$

Suggestion: The integration formula obtained in Exercise 1, Sec. 79, is needed here.