Exercise 14

Let $F(s)$ be the function in Exercise 13, and write the nonzero coefficient b_m there in exponential form as $b_m = r_m \exp(i\theta_m)$. Then use the main result in part (b) of Exercise 13 to show that when t is real, the sum of the residues of $e^{st}F(s)$ at $s_0 = \alpha + i\beta$ $(\beta \neq 0)$ and $\overline{s_0}$ contains a term of the type

$$\frac{2r_m}{(m-1)!} t^{m-1} e^{\alpha t} \cos(\beta t + \theta_m).$$

Note that if $\alpha > 0$, the product $t^{m-1}e^{\alpha t}$ here tends to ∞ as t tends to ∞. When the inverse Laplace transform $f(t)$ is found by summing the residues of $e^{st}F(s)$, the term displayed just above is, therefore, an *unstable* component of $f(t)$ if $\alpha > 0$; and it is said to be of *resonance* type. If $m \geq 2$ and $\alpha = 0$, the term is also of resonance type.