

Exercise 6

Verify

- (a) the associative law for addition of complex numbers, stated at the beginning of Sec. 2;
- (b) the distributive law (3), Sec. 2.

Solution

Part (a)

The associative law for addition of complex numbers states that

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3).$$

To verify it, let $z_1 = 6i$ and $z_2 = -1 + 2i$ and $z_3 = 2 - 4i$. Check to see whether both sides are equal.

$$\begin{aligned} [(6i) + (-1 + 2i)] + (2 - 4i) &\stackrel{?}{=} (6i) + [(-1 + 2i) + (2 - 4i)] \\ [-1 + 8i] + 2 - 4i &\stackrel{?}{=} 6i + [1 - 2i] \\ 1 + 4i &= 1 + 4i \end{aligned}$$

The associative law for addition is verified.

Part (b)

The distributive law states that

$$z(z_1 + z_2) = zz_1 + zz_2.$$

To verify it, let z_1 and z_2 be the same values as before and let $z = 3 + 5i$. Check to see whether both sides are equal.

$$\begin{aligned} (3 + 5i)[(6i) + (-1 + 2i)] &\stackrel{?}{=} (3 + 5i)(6i) + (3 + 5i)(-1 + 2i) \\ (3 + 5i)[-1 + 8i] &\stackrel{?}{=} (18i + 30i^2) + (-3 + 6i - 5i + 10i^2) \\ -3 + 24i - 5i + 40i^2 &\stackrel{?}{=} 18i - 30 - 3 + i - 10 \\ -3 + 19i - 40 &\stackrel{?}{=} 19i - 43 \\ -43 + 19i &= -43 + 19i \end{aligned}$$

The distributive law is verified.