Exercise 11

Use mathematical induction to show that when $n = 2, 3, \ldots,$

$$(a)\ \overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}; \qquad (b)\ \overline{z_1 z_2 \cdots z_n} = \overline{z_1}\,\overline{z_2} \cdots \overline{z_n}.$$

Solution

Part (a)

Start by showing that the result holds in the base case $n = 2$.

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

This is property (2) in the text, which has been shown to be true. Now assume the inductive hypothesis,

$$\overline{z_1 + z_2 + \cdots + z_k} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_k},$$

and show that

$$\overline{z_1 + z_2 + \cdots + z_k + z_{k+1}} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_k} + \overline{z_{k+1}}.$$

Do so by grouping the first k terms, using the base case, and then using the inductive hypothesis.

$$\begin{aligned}\overline{z_1 + z_2 + \cdots + z_k + z_{k+1}} &= \overline{(z_1 + z_2 + \cdots + z_k) + z_{k+1}} \\ &= \overline{z_1 + z_2 + \cdots + z_k} + \overline{z_{k+1}} \\ &= \overline{z_1} + \overline{z_2} + \cdots + \overline{z_k} + \overline{z_{k+1}}\end{aligned}$$

Therefore, by mathematical induction,

$$\overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}.$$

Part (b)

Start by showing that the result holds in the base case $n = 2$.

$$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$$

This is property (4) in the text, which has been shown to be true. Now assume the inductive hypothesis,

$$\overline{z_1 z_2 \cdots z_k} = \overline{z_1}\,\overline{z_2} \cdots \overline{z_k},$$

and show that

$$\overline{z_1 z_2 \cdots z_k z_{k+1}} = \overline{z_1}\,\overline{z_2} \cdots \overline{z_k}\,\overline{z_{k+1}}.$$

Do so by grouping the first k terms, using the base case, and then using the inductive hypothesis.

$$\begin{aligned}\overline{z_1 z_2 \cdots z_k z_{k+1}} &= \overline{(z_1 z_2 \cdots z_k) z_{k+1}} \\ &= \overline{z_1 z_2 \cdots z_k}\,\overline{z_{k+1}} \\ &= \overline{z_1}\,\overline{z_2} \cdots \overline{z_k}\,\overline{z_{k+1}}\end{aligned}$$

Therefore, by mathematical induction,

$$\overline{z_1 z_2 \cdots z_n} = \overline{z_1}\,\overline{z_2} \cdots \overline{z_n}.$$