

Exercise 7

Use established properties of moduli to show that when $|z_3| \neq |z_4|$,

$$\left| \frac{z_1 + z_2}{z_3 + z_4} \right| \leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||}.$$

Solution

Apply the triangle inequality twice—once in the numerator to make it as big as possible and once in the denominator to make it as small as possible.

$$\begin{aligned} \left| \frac{z_1 + z_2}{z_3 + z_4} \right| &= \frac{|z_1 + z_2|}{|z_3 + z_4|} \\ &\leq \frac{|z_1| + |z_2|}{|z_3 + z_4|} \\ &\leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||} \end{aligned}$$