

Exercise 4

Verify that each of the two numbers $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$.

Solution

Substitute the two numbers into the equation and check that the equation is satisfied. Check $z = 1 + i$ first.

$$\begin{aligned}0 &\stackrel{?}{=} (1 + i)^2 - 2(1 + i) + 2 \\ &\stackrel{?}{=} (1 + 2i + i^2) - (2 + 2i) + 2 \\ &\stackrel{?}{=} 1 + 2i - 1 - 2 - 2i + 2 \\ &\stackrel{?}{=} 0 + 0i \\ &= 0\end{aligned}$$

$z = 1 + i$ is indeed a solution of the equation. Check $z = 1 - i$ now.

$$\begin{aligned}0 &\stackrel{?}{=} (1 - i)^2 - 2(1 - i) + 2 \\ &\stackrel{?}{=} (1 - 2i + i^2) - (2 - 2i) + 2 \\ &\stackrel{?}{=} 1 - 2i - 1 - 2 + 2i + 2 \\ &\stackrel{?}{=} 0 + 0i \\ &= 0\end{aligned}$$

$z = 1 - i$ is indeed a solution of the equation.