

Exercise 5

Prove that multiplication of complex numbers is commutative, as stated at the beginning of Sec. 2.

Solution

Here we have to show that

$$z_1 z_2 = z_2 z_1,$$

where z_1 and z_2 are complex numbers. Use the definition of multiplication of two complex numbers, equation (4) on page 2.

$$\begin{aligned} z_1 z_2 &= (x_1, y_1)(x_2, y_2) \\ &= (x_1 x_2 - y_1 y_2, y_1 x_2 + x_1 y_2) \end{aligned}$$

Use the commutative property of addition for real numbers.

$$z_1 z_2 = (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$$

Use the commutative property of multiplication for real numbers.

$$\begin{aligned} z_1 z_2 &= (x_2 x_1 - y_2 y_1, y_2 x_1 + x_2 y_1) \\ &= (x_2, y_2)(x_1, y_1) \\ &= z_2 z_1 \end{aligned}$$