

Exercise 5

Derive expression (6), Sec. 3, for the quotient z_1/z_2 by the method described just after it.

Solution

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ and assume $x_1, x_2, y_1,$ and y_2 are real numbers.

$$\begin{aligned}\frac{z_1}{z_2} &= \frac{x_1 + iy_1}{x_2 + iy_2} \\ &= \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} \\ &= \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 - i^2y_2^2} \\ &= \frac{x_1x_2 - ix_1y_2 + ix_2y_1 - i^2y_1y_2}{x_2^2 + y_2^2} \\ &= \frac{x_1x_2 + y_1y_2 + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2} \\ &= (x_2^2 + y_2^2)^{-1}[x_1x_2 + y_1y_2 + i(x_2y_1 - x_1y_2)] \\ &= (x_2^2 + y_2^2)^{-1}(x_1x_2 + y_1y_2) + i(x_2^2 + y_2^2)^{-1}(x_2y_1 - x_1y_2) \\ &= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}\end{aligned}\tag{6}$$

This is Equation (6) on page 6.