Exercise 1

Use properties of conjugates and moduli established in Sec. 6 to show that

$$(a)\ \overline{\bar{z}+3i} = z-3i; \qquad (b)\ \overline{iz} = -i\bar{z};$$
$$(c)\ \overline{(2+i)^2} = 3-4i; \qquad (d)\ |(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}|2z+5|.$$

Solution

Part (a)

Use the fact that the conjugate of a sum is the sum of the conjugates.

$$\begin{aligned}\overline{\bar{z}+3i} &= \bar{\bar{z}} + \overline{3i} \\ &= (z) + (-3i) \\ &= z - 3i\end{aligned}$$

Part (b)

Use the fact that the conjugate of a product is the product of the conjugates.

$$\begin{aligned}\overline{iz} &= \bar{i}\bar{z} \\ &= (-i)\bar{z} \\ &= -i\bar{z}\end{aligned}$$

Part (c)

Use the fact that the conjugate of a product is the product of the conjugates.

$$\begin{aligned}\overline{(2+i)^2} &= \overline{(2+i)(2+i)} \\ &= \overline{2+i}\,\overline{2+i} \\ &= (2-i)(2-i) \\ &= 4-4i+i^2 \\ &= 3-4i\end{aligned}$$

Part (d)

Use the fact that the modulus of a complex number is equal to the modulus of its conjugate.

$$\begin{aligned}|(2\bar{z}+5)(\sqrt{2}-i)| &= \left|\overline{2z+5}\,\overline{\sqrt{2}+i}\right| \\ &= |\overline{2z+5}|\left|\overline{\sqrt{2}+i}\right| \\ &= |2z+5||\sqrt{2}+i| \\ &= |2z+5|\sqrt{2+1} \\ &= \sqrt{3}|2z+5|\end{aligned}$$