

Exercise 5

Verify property (9) of moduli in Sec. 6.

Solution

Property (9) in Sec. 6 is on page 15.

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, \quad z_2 \neq 0 \quad (9)$$

Start with the relationship between the conjugate and the modulus of a complex number.

$$\begin{aligned} \left| \frac{z_1}{z_2} \right|^2 &= \left(\frac{z_1}{z_2} \right) \overline{\left(\frac{z_1}{z_2} \right)} \\ &= \left(\frac{z_1}{z_2} \right) \left(\frac{\bar{z}_1}{\bar{z}_2} \right) \\ &= \frac{z_1 \bar{z}_1}{z_2 \bar{z}_2} \\ &= \frac{|z_1|^2}{|z_2|^2} \\ &= \left(\frac{|z_1|}{|z_2|} \right)^2 \end{aligned}$$

Therefore,

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|},$$

provided that $z_2 \neq 0$.