

## Exercise 8

It is shown in Sec. 3 that if  $z_1 z_2 = 0$ , then at least one of the numbers  $z_1$  and  $z_2$  must be zero. Give an alternative proof based on the corresponding result for real numbers and using identity (8), Sec. 6.

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### Solution

Suppose that  $z_1 z_2 = 0$ . Then  $|z_1 z_2| = 0$ . Then  $|z_1| |z_2| = 0$  by identity (8).  $|z_1|$  and  $|z_2|$  are real numbers, so  $|z_1| = 0$  or  $|z_2| = 0$  by the zero product property. The only number with a magnitude of zero is zero, which means  $z_1 = 0$  or  $z_2 = 0$ . Therefore, if  $z_1 z_2 = 0$ , then  $z_1 = 0$  or  $z_2 = 0$ .